

# Channel Sensing and Communication over a Time-Correlated Channel with an Energy Harvesting Transmitter

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## Abstract

We consider an energy harvesting (EH) transmitter communicating over a time-correlated wireless channel. The transmitter is capable of sensing the current channel state, albeit at the cost of both energy and transmission time. The EH transmitter aims to maximize its long-term throughput by choosing one of the following actions: *i*) defer its transmission to save energy for future use, *ii*) transmit reliably at a low rate, *iii*) transmit at a high rate, and *iv*) sense the channel to reveal the channel state information at a cost of energy and transmission time, and then decide to defer or to transmit. The problem is formulated as a partially observable Markov decision process with a belief on the channel state. The optimal policy is shown to exhibit a threshold behavior on the belief state, with battery-dependent threshold values. The optimal threshold values and performance are characterized numerically via the value iteration algorithm. Our results demonstrate that, despite the associated time and energy cost, sensing the channel intelligently to track the channel state improves the achievable long-term throughput significantly as compared to the performance of those protocols lacking this ability as well as the one that always senses the channel.

## Index Terms

Gilbert-Elliot channel, channel sensing, Markov decision process.

## I. INTRODUCTION

Due to the tremendous increase in the number of battery-powered wireless communication devices over the past decade, replenishing the batteries of these devices by harvesting energy from

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natural resources has become an important research area [1]. Transmitters may harvest energy via wind turbines, photovoltaic cells, thermoelectric generators, or from mechanical vibrations through piezoelectric or electromagnetic technology [2]. Regardless of which type of energy harvesting (EH) device and natural energy source is employed, a main concern is the stochastic nature of the EH process driving the wireless communications. The associated battery recharging process can be modeled either as a continuous, or a discrete stochastic process [3], [4], [5], [6].

We consider a wireless point-to-point channel, and a transmitter equipped with a finite-capacity battery fed by an EH device. The time is divided into discrete time slots, and at each time slot, a unit of energy is harvested by the transmitter according to a binary random process independent over time<sup>1</sup>. We assume that the transmitter can accurately observe the current energy level of the battery, and it has the knowledge of the statistics of the EH process. The wireless channel is time-varying and has memory across time. The channel memory is modeled with a finite state Markov chain [7], where the channel state in the next time slot depends only on the current channel state. A convenient and often-employed simplification of the Markov model is a two-state Markov chain, known as the Gilbert-Elliot channel [8]. This model assumes that the channel can be either in a *GOOD* or a *BAD* state. We assume that, in a *GOOD* state, by spending exactly one unit of energy from its battery, the transmitter can transmit  $R_2$  bits of information within a time slot, while in a *BAD* state, it can only transmit  $R_1$  bits, where  $R_1 < R_2$ .

In this work, differently from most of the literature on EH systems, we take into account the energy cost of acquiring channel state information (CSI). At the beginning of each time slot, without knowing the current CSI, EH transmitter takes one of the following actions: *i*) defer the transmission to save its energy for future use; *ii*) transmit at a low rate of  $R_1$  bits while guaranteeing successful delivery; *iii*) transmit at a high rate of  $R_2$  bits and risk an unsuccessful transmission if the channel is in a *BAD* state, and *iv*) sensing the channel to reveal the channel state with a time and energy cost, and then deciding either to defer or transmit at a rate according to the revealed channel state. Our objective is to maximize the expected discounted sum of bits transmitted over an infinite time horizon.

<sup>1</sup>Typically, the EH process is neither memoryless nor discrete, and the energy is accumulated continuously over time. However, in order to develop the analytical model underlying this paper, we follow the common assumption in the literature [3], and assume that the continuous energy arrival is accumulated in an intermediate energy storage device to form energy quantas.

### A. Related Work

Markov decision process (MDP) tools have been extensively utilized in the literature to model the communication systems with EH devices. In [9], the authors propose a simple single-threshold policy for a solar-powered sensor operating over a fading wireless channel. The optimality of a single-threshold policy is proven in [10] when an EH transmitter transmits packets with varying importance. The allocation of energy for collecting and transmitting data in an EH communication system is studied in [11] and [12]. The scheduling of EH transmitters with time correlated energy arrivals to optimize the long term sum throughput is investigated in [13]. Meanwhile, the problem of allocating of energy over a finite time horizon to optimize the throughput is addressed in [14], when either the current or future energy and channel states are known by the transmitter. In [15], power allocation to maximize the throughput is studied when the amount of harvested energy and the channel state are modeled as Markov and static processes, respectively. Finally, in [16], a threshold based transmission scheme over a multiple access channel with no feedback was investigated when the EH processes are spatially correlated.

Gilbert-Elliott model has been previously investigated in the context of scheduling an EH transmitter in [17], where the transmitter always has perfect CSI, obtained by sensing at every time slot. The transmitter makes a decision to defer or to transmit based on the current CSI and the battery state. Similarly, without considering the channel sensing capability, [18] addresses the problem of optimal power management for an EH sensor over a multi-state wireless channel with memory. Unlike previous work, we take into account the energy cost of channel sensing which can be significant for a low-power EH transmitter. Therefore, in order to minimize the energy consumed for channel sensing, an EH transmitter does not necessarily sense the channel at every time slot, and instead, it keeps an updated belief of the channel state according to its past observations, and only occasionally senses the current channel state.

Channel sensing is an essential part of opportunistic and cognitive spectrum access. In [19], the authors investigate the problem of optimal access to a Gilbert-Elliott channel, wherein an energy-unlimited transmitter senses the channel at every time slot. In [20] channel sensing is done only occasionally. The transmitter can decide to transmit at a high or a low rate without sensing the channel, or it can first sense the channel and transmit at a reduced rate due to the time spent for sensing. However, the energy cost of sensing is ignored in [20].

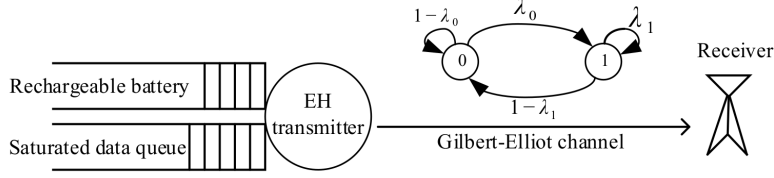


Fig. 1: System model.

### B. Organization of the paper

In Section II, we explain the channel and EH processes, and elaborate on the transmission protocol. In Section III, we formulate the problem as a two-state partially observable MDP (POMDP) which is then converted to a continuous-state MDP by introducing a belief state. In Section IV, we show that the optimal policy is of threshold type, for which the optimal threshold values on the belief state depend on the state of the battery. In Section V, we present the results of our Monte-Carlo simulations where we numerically obtain the optimal threshold values and the corresponding optimal performance. Finally, we conclude the paper and present future research directions in Section VI.

## II. SYSTEM MODEL

### A. Channel and energy harvesting models

Consider the communication system illustrated in Fig. 1, where an EH transmitter communicates over a slotted Gilbert-Elliot channel. Let  $G_t$  denote the state of the channel at time slot  $t$ , which is modeled as a one-dimensional Markov chain with two states: GOOD state denoted by 1, and BAD state denoted by 0. Channel transitions occur at the beginning of each time slot. The transition probabilities are given by  $\mathbb{P}[G_t = 1|G_{t-1} = 1] = \lambda_1$  and  $\mathbb{P}[G_t = 1|G_{t-1} = 0] = \lambda_0$ . We assume that the states are positively correlated, i.e.,  $\lambda_0 \leq \lambda_1$  as described in [21]. We consider a simple constant-power transmitter which can employ error correcting codes at two different rates, each designed to achieve (almost) reliable transmission at one of the channel states. Accordingly, the transmitter is able to transmit  $R_2$  bits per time slot if  $G_t = 1$ , and  $R_1 < R_2$  bits if  $G_t = 0$ . We normalize the slot duration to one unit and hence  $R_1$  and  $R_2$  refer to both the transmission rate and the number of transmitted bits in a time slot. We assume that the transmitter has an infinitely backlogged data queue, and thus, it always has data to transmit.

A unit of energy arrives at the end of time slot  $t$  according to an independent and identically distributed (i.i.d.) Bernoulli process<sup>2</sup>, denoted by  $E_t$ , with probability  $q$ , i.e.,  $\mathbb{P}[E_t = 1] = q$  for all  $t$ . The transmitter stores the energy packets in a battery with a storage capacity of  $B_{max}$  units of energy. We denote the state of the battery, i.e., the energy available in the battery at the beginning of time slot  $t$ , by  $B_t$ .

### B. Transmission protocol

Once a transmission at a rate of  $R_2$  occurs, either in a GOOD or a BAD state, the receiver replies with an acknowledgment (ACK) if the transmission is successful, or with a negative acknowledgment (NACK) if the transmission fails. Note that an ACK message informs the transmitter that the most recent state of the channel was GOOD, whereas a NACK message informs otherwise.

At the beginning of each time slot, the transmitter takes one of the following actions: *i*) defer the transmission, *ii*) transmit at rate  $R_1$ , *iii*) transmit at rate  $R_2$ , and *iv*) sense the channel and transmit or defer, based on the channel state.

*i) Defer transmission:* This action (denoted by  $D$ ) corresponds to the case when the transmitter remains idle. It helps the transmitter prevent possible future energy outages which would otherwise disable the transmitter send any data, even if the channel is in a GOOD state. If this action is chosen, there is no message exchange between the transmitter and the receiver. Hence, the transmitter does not obtain the current CSI<sup>3</sup>.

*ii) Transmit at rate  $R_1$ :* This action (denoted by  $L$ ) corresponds to making a transmission at rate  $R_1$  without sensing the channel. If this action is chosen, the transmitter uses a high redundancy coding scheme to guarantee the successful delivery of the message. Since the delivery of the information is guaranteed, the receiver does not send any feedback, and thus, the transmitter does not obtain the current CSI.

*iii) Transmit at rate  $R_2$ :* This action (denoted by  $H$ ) corresponds to transmitting at rate  $R_2$  without sensing the channel. If the channel is in a GOOD state, the transmission is successful and

<sup>2</sup>There is an enormous body of the literature (see for example [17], [22], and references therein) which assumes i.i.d. EH processes. Nevertheless, results presented in this work can be easily extended to consider time-correlated EH processes. We chose to restrict our attention to i.i.d. EH processes for the clarity of the exposition.

<sup>3</sup>The scenario in which the transmitter is informed about the current CSI even when it does not transmit any data packet is equivalent to the system model investigated in [17].

the receiver sends an ACK. Otherwise, the transmission fails, and the receiver sends a NACK. The perfect feedback from the receiver allows the transmitter to obtain the exact CSI for the completed time slot.

*iv) Channel sensing:* The transmitter decides to sense the channel at the beginning of the time slot. The sensing consumes a fraction  $0 < \tau < 1$  of an energy unit. The sensing operation is carried out by the transmitter first sending a control/probing packet, to which, the receiver responds with a response packet indicating the channel state. We assume that the sensing takes  $\tau$  portion of a time slot and the transmitter consumes on average the same power as data transmission over the sensing period. For simplicity let  $\tau = 1/k$  for some  $k \in \mathbb{Z}^+$ . After sensing the channel, in the remaining  $1 - \tau$  portion of a time slot, the transmitter may choose to defer transmission, or to transmit data at a rate  $R_1$  or  $R_2$  depending on the revealed channel state. However, since the transmission spans only  $1 - \tau$  seconds, a total of  $(1 - \tau)R_i$  bits,  $i = 1, 2$ , are transmitted by the end of the time slot.

If the channel is revealed to be in a BAD state, the transmitter has two options; either defer transmission (denoted by  $OD$ ) and save the rest of the energy unit (i.e.,  $1 - \tau$ ), or utilize the rest of the energy unit to transmit at a lower rate,  $R_1$  (denoted by  $OT$ ). On the other hand, in a GOOD channel state, the transmitter always transmits at rate  $R_2$  in the remainder of the time slot. Note that, thanks to the channel sensing capability, the transmitter can adapt its behavior to the current channel state. As we will show later in this paper, this proves to be an important capability to improve the efficiency in EH networks with scarce energy sources.

### III. PARTIALLY OBSERVABLE MARKOV DECISION PROCESS (POMDP) FORMULATION

At the beginning of each time slot, the transmitter chooses an action among the five possible actions in its action set  $\mathcal{A} \triangleq \{D, L, OD, OT, H\}$ , based on the state of its battery and its belief about the channel state to maximize a long-term discounted reward to be defined shortly. Although the transmitter is perfectly aware of its battery state, it does not always observe the current channel state. Hence, the problem can be formulated as a partially observable Markov decision process (POMDP).

Let the state of the system at time  $t$  be denoted by  $S_t = (B_t, X_t)$ . We define the *belief* of the transmitter at time slot  $t$ , denoted by  $X_t$ , as the conditional probability that the channel is in a GOOD state at the beginning of the current slot, given the history  $\mathcal{H}_t$ , i.e.,  $X_t = \mathbb{P}[G_t = 1 | \mathcal{H}_t]$ , where  $\mathcal{H}_t$  represents all past actions and observations of the transmitter up to, but not including

slot  $t$ . The belief of transmitter constitutes a sufficient statistic to characterize its optimal actions [23]. Note that with this definition of a state, the POMDP problem is converted into a MDP with an uncountable state space<sup>4</sup>  $\{0, \tau, 2\tau, \dots, B_{max}\} \times [0, 1]$ .

A transmission policy  $\pi$  describes a set of rules that dictates which action to take at each slot depending on the history. Let  $V^\pi(b, p)$  be the expected infinite-horizon discounted reward with initial state  $S_0 = (b, \mathbb{P}[G_0 = 1 | \mathcal{H}_0] = p)$  under policy  $\pi$  with discount factor  $\beta \in [0, 1]$ . The use of the expected discounted reward allows us to obtain a tractable solution, and one can gain insights into the optimal policy for the average reward when  $\beta$  is close to 1. It is also discussed in [5] that  $\beta$  can be interpreted as the probability that a particular user is allowed to use the channel, or as the probability of the transmitter to remain active at each time slot as in [24]. For an initial belief  $p$ , the expected discounted reward has the following expression

$$V^\pi(b, p) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t R(S_t, A_t) | S_0 = (b, p) \right], \quad (1)$$

where  $t$  is the time index,  $A_t \in \mathcal{A}$  is the action chosen at time  $t$ , and  $R(S_t, A_t)$  is the expected reward acquired when action  $A_t$  is taken at state  $S_t$ . The expectation in (1) is over state sequence distribution induced by the given transmission policy  $\pi$ . The expected reward when action  $A_t$  is chosen at state  $S_t$  is given as follows:

$$R(S_t, A_t) = \begin{cases} X_t R_2, & \text{if } A_t = H \text{ and } B_t \geq 1, \\ R_1, & \text{if } A_t = L \text{ and } B_t \geq 1, \\ (1 - \tau) X_t R_2, & \text{if } A_t = OD \text{ and } B_t \geq 1, \\ (1 - \tau)[(1 - X_t) R_1 + X_t R_2], & \text{if } A_t = OT \text{ and } B_t \geq 1, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Since at least one energy unit is required for transmission, if the battery state is less than one unit, the reward becomes zero. Hence, in (2), we consider actions are taken when the battery state is greater than or equal to one. If the action of transmitting at rate  $R_2$  without sensing is chosen,  $R_2$  bits are transmitted successfully if the channel is in a GOOD state, and 0 bits if the channel is in a BAD state. Since the belief,  $X_t$ , represents the probability of the channel being in a GOOD state, the expected reward is given by  $X_t R_2$ . It is guaranteed that transmitting at low rate is always successful so the expected reward for this action is  $R_1$ . If the action of channel

<sup>4</sup>Note that since sensing without transmission is possible, i.e., consuming only  $\tau$  fraction of the energy unit, the battery can take fraction of units as states.

sensing is chosen,  $\tau$  fraction of energy unit is spent sensing the channel with the remaining energy either being used for transmission or saved in the battery. If the channel is sensed to be in a GOOD state,  $(1 - \tau)R_2$  bits are transmitted successfully. If the channel is sensed to be in a BAD state, the transmitter either remains silent by saving the rest of the energy unit and receives no rewards, or utilizes the rest of the energy unit and transmits  $(1 - \tau)R_1$  in the rest of the time slot. Thus, the expected reward by choosing action  $OD$  is  $(1 - \tau)X_t R_2$ , while the expected reward by choosing  $OT$  is  $(1 - \tau)[(1 - X_t)R_1 + X_t R_2]$ . Finally, if the action of deferring the transmission is taken, the transmitter neither senses the channel nor transmits data, so the reward is zero.

Define the value function  $V(b, p)$  as

$$V(b, p) = \max_{\pi} V^{\pi}(b, p) \quad \text{for all } b \in \{0, \tau, 2\tau, \dots, B_{max}\} \text{ and } p \in [0, 1]. \quad (3)$$

It is well known that the optimal value of the infinite-horizon expected reward can be achieved by a stationary policy, i.e., there exists a stationary policy  $\pi^*$ , mapping the state space  $\{0, \tau, 2\tau, \dots, B_{max}\} \times [0, 1]$  into the action space  $\mathcal{A}$ , such that  $V(b, p) = V^{\pi^*}(b, p)$  [25]. The value function  $V(b, p)$  satisfies the Bellman equation

$$V(b, p) = \max_{A \in \{D, L, OD, OT, H\}} \{V_A(b, p)\}, \quad (4)$$

where  $V_A(b, p)$  is the action-value function, defined as the expected infinite-horizon discounted reward acquired by taking action  $A$  when the state is  $(b, p)$ , and is given by

$$V_A(b, p) = R((b, p), A) + \beta \mathbb{E}_{(\acute{b}, \acute{p})} \left[ V(\acute{b}, \acute{p}) | S_0 = (b, p), A_0 = A \right], \quad (5)$$

where  $(\acute{b}, \acute{p})$  denotes the next state when action  $A$  is chosen at state  $S_0 = (b, p)$ . The expectation in (5) is over the distribution of possible next states. In the following, we define and explain the value function  $V_A(b, p)$ , and how the system state evolves for each action.

*Defer transmission:* If this action is taken, since there is no transmission, there is no ACK or NACK from the receiver, and thus, the transmitter does not learn the state of the channel. Therefore, the next belief is obtained as the probability of finding the channel in a GOOD state given the current belief state. If the transmitter had a belief  $X_t = p$  at time slot  $t$ , after taking action  $D$ , its belief at the beginning of the next slot is updated as

$$J(p) = \lambda_0(1 - p) + \lambda_1 p. \quad (6)$$



In every time slot, a unit of energy is harvested with probability  $q$ . Thus, after taking action  $D$ , the value function evolves as follows:

$$V_D(b, p) = \beta [qV(\min\{b+1, B_{max}\}, J(p)) + (1-q)V(b, J(p))]. \quad (7)$$

Note that the term  $\min\{b+1, B_{max}\}$  is used to ensure that the battery state does not exceed the battery capacity,  $B_{max}$ .

*Transmit at rate  $R_1$ :* Since each transmission consumes one unit of energy, this action can be taken only if  $b \geq 1$ <sup>5</sup>. If this action is taken, the transmission will be successful independent of the channel state. Hence, there is no ACK or NACK from the receiver, and as a result the transmitter does not learn the state of the channel. Similar to action  $D$ , the next belief of the channel state is updated as  $J(p) = \lambda_0(1-p) + \lambda_1 p$ , and the value function corresponding to this action is updated as follows:

$$V_L(b, p) = R_1 + \beta [qV(b, J(p)) + (1-q)V(b-1, J(p))]. \quad (8)$$

*Transmit at rate  $R_2$ :* This action can only be chosen if  $b \geq 1$ . If the channel is in GOOD state,  $R_2$  bits are successfully delivered to the receiver, the receiver sends back an ACK, and the belief for the next time slot is updated as  $\lambda_1$ . Otherwise, the transmission fails, the receiver sends a NACK, and the belief is updated  $\lambda_0$ . Hence, the value function evolves as:

$$\begin{aligned} V_H(b, p) = & p[R_2 + \beta(qV(b, \lambda_1) + (1-q)V(b-1, \lambda_1))] \\ & + (1-p)\beta[qV(b, \lambda_0) + (1-q)V(b-1, \lambda_0)] \end{aligned} \quad (9)$$

*Channel sensing:* For this action, depending on the battery state, two scenarios are possible. If  $b \geq 1$  and EH decides to sense the channel, then it consumes  $\tau$  fraction of energy to first sense the channel and obtain the current channel state. Based on the outcome of the channel sensing, if the channel is found to be in a GOOD state,  $(1-\tau)$  units of energy is used to transmit  $(1-\tau)R_2$  bits, and the belief state is updated as  $\lambda_1$  for the next time slot. Note that the transmitter always transmits if the channel is in a GOOD state, because this is the best state possible and saving energy for future cannot improve the reward.

<sup>5</sup>Note that in the generic MDP formulation, we have the same set of actions in every state. We can re-define the reward function by assigning  $-\infty$  reward to those actions that are not possible to be taken in specific states to account for this. For the ease of exposition, we chose to present the formulation in this manner.

On the other hand, if the outcome of the channel sensing reveals the channel to be in a BAD state, then the transmitter either defers its transmission (takes action  $OD$ ), and saves  $(1 - \tau)$  units of energy for possible future transmissions, or utilizes  $(1 - \tau)$  units of energy and transmits  $(1 - \tau)R_1$  bits (takes action  $OT$ ). The channel belief is updated as  $\lambda_0$  for the next time slot. Based on the aforementioned discussion, for  $b \geq 1$ , the evolution of the value function is given as:

$$\begin{aligned} V_{OD}(b, p) = & p[(1 - \tau)R_2 + \beta(qV(b, \lambda_1) + (1 - q)V(b - 1, \lambda_1))] \\ & + (1 - p)\beta[qV(\min\{b - \tau + 1, B_{max}\}, \lambda_0) + (1 - q)V(b - \tau, \lambda_0)]. \end{aligned} \quad (10)$$

$$\begin{aligned} V_{OT}(b, p) = & p[(1 - \tau)R_2 + \beta(qV(b, \lambda_1) + (1 - q)V(b - 1, \lambda_1))] \\ & + (1 - p)[(1 - \tau)R_1 + \beta(qV(b, \lambda_0) + (1 - q)V(b - 1, \lambda_0))] \end{aligned} \quad (11)$$

Meanwhile, if  $\tau \leq b < 1$ , then transmission is not possible since it requires at least one unit of energy. However, it is still possible to sense the channel, since it only requires  $\tau$  fraction of energy. This case may arise when EH node believes that learning the channel state may help its decision in the future. Thus for  $\tau \leq b < 1$ , the value function evolves as:

$$\begin{aligned} V_{OD}(b, p) = & \beta[qpV(b - \tau + 1, \lambda_1) \\ & + q(1 - p)V(b - \tau + 1, \lambda_0) + (1 - q)pV(b - \tau, \lambda_1) \\ & + (1 - q)(1 - p)V(b - \tau, \lambda_0)] \end{aligned} \quad (12)$$

#### IV. THE STRUCTURE OF THE OPTIMAL POLICY

##### A. General Case

In this section, we show that the optimal policy has a threshold-type structure on the belief state. The belief state set, i.e., the interval  $[0, 1]$  can be divided into mutually exclusive subsets where each subset is assigned to an action. We begin to establish our main results by proving the convexity of the value function  $V(b, p)$ , with respect to  $p$ .

**Lemma 1.** *For any given  $b \geq 0$ ,  $V(b, p)$  is convex in  $p$ .*

*Proof.* The proof is given in Appendix A. □

In the following, we show that the value function is a non-decreasing function of the battery state,  $b$ . This lemma provides the intuition why deferring or sensing actions are advantageous

in some states. The incentive of taking these actions is that the value function transitions into higher values without consuming any energy, or consuming only  $\tau$  fraction of an energy unit.

**Lemma 2.** *Given an arbitrary belief  $0 \leq p \leq 1$ ,  $V(b_1, p) \geq V(b_0, p)$  if  $b_1 > b_0$ .*

*Proof.* The proof is given in Appendix B.  $\square$

The next result states that the value function is also non-decreasing with respect to the belief state,  $p$ .

**Lemma 3.** *For a given battery state  $b \in \{0, \tau, 2\tau, \dots, B_{max}\}$ , if  $p_1 > p_0$  then  $V(b, p_1) \geq V(b, p_0)$ .*

*Proof.* The proof is given in Appendix C.  $\square$

Lemma 1 is necessary in proving the structure of the optimal policy. For each  $b \in \{0, \tau, 2\tau, \dots, B_{max}\}$ , we define the following sets:

$$\Phi_A^b \triangleq \{p \in [0, 1] : V(b, p) = V_A(b, p)\}, \text{ for } A \in \mathcal{A}. \quad (13)$$

Note that, given any battery state  $b \geq 0$ ,  $\Phi_A^b$  characterizes the set of belief states for which it is optimal to choose action  $A \in \mathcal{A}$ . In Theorem 1, we show that the optimal policy has a threshold-type structure.

**Theorem 1.** *For given set of actions  $\mathcal{A} = \{D, L, OD, OT, H\}$ , the optimal policy is a threshold-type policy on the belief state  $p$ , in which, the thresholds are functions of the battery state,  $b$ .*

*Proof.* This lemma proves that the optimal policy has a threshold structure. Consider  $\Phi_A^b$  for  $A \in \{D, L, OD, OT, H\}$ . Initially, we aim to prove that  $\Phi_A^b$  for  $A \in \{OD, OT, H\}$  is convex. It is easy to see that for  $b = 0$ ,  $V(b, p) = V_D(b, p)$ , and hence,  $\Phi_D^0 = [0, 1]$ , and  $\Phi_L^0 = \Phi_{OD}^0 = \Phi_{OT}^0 = \Phi_H^0 = \emptyset$ . First, we consider battery states  $\tau \leq b < 1$ . We will prove that for any  $\tau \leq b < 1$ ,  $\Phi_{OD}^b$  is convex. Let  $p_1, p_2 \in \Phi_{OD}^b$ , and  $a \in (0, 1)$ . We have

$$V(b, ap_1 + (1 - a)p_2) \leq aV(b, p_1) + (1 - a)V(b, p_2), \quad (14)$$

$$= aV_{OD}(b, p_1) + (1 - a)V_{OD}(b, p_2), \quad (15)$$

$$= V_{OD}(b, ap_1 + (1 - a)p_2), \quad (16)$$

$$\leq V(b, ap_1 + (1 - a)p_2), \quad (17)$$

where (14) follows from Lemma 1; (15) is due to the fact that  $p_1, p_2 \in \Phi_{OD}^b$ ; (16) follows from the linearity of  $V_{OD}$  in  $p$ ; and (17) holds due to the definition of  $V(b, p)$ . Consequently,  $V(b, ap_1 + (1-a)p_2) = V_{OD}(b, ap_1 + (1-a)p_2)$ , and it follows that  $ap_1 + (1-a)p_2 \in \Phi_{OD}^b$ , which, in turn, proves the convexity of  $\Phi_{OD}^b$ . Note also that  $p = 0$  and  $p = 1$  both belong to  $\Phi_D^b$  for all  $0 \leq b < 1$ . Since no transmission is possible for  $0 \leq b < 1$ , we have  $\Phi_L^b = \Phi_H^b = \emptyset$ . Hence, for a given battery state  $0 \leq b < 1$ , either  $\Phi_{OD}^b = \emptyset$ , or there exists  $0 < \rho_1(b) \leq \rho_2(b) < 1$  such that  $\Phi_{OD}^b = [\rho_1(b), \rho_2(b)]$ . Consequently, we have  $\Phi_D^b = [0, \rho_1(b)) \cup (\rho_2(b), 1]$ , if  $0 \leq b < 1$ .

Next, consider  $1 \leq b \leq B_{max}$ . We will prove that  $\Phi_H^b$ ,  $\Phi_{OD}^b$ , and  $\Phi_{OT}^b$  are convex subsets of the belief state set. Let  $p_1, p_2 \in \Phi_H^b$  and  $a \in (0, 1)$ . Similar to (14)-(17) we can argue

$$\begin{aligned} V(b, ap_1 + (1-a)p_2) &\leq aV(b, p_1) + (1-a)V(b, p_2), \\ &= aV_H(b, p_1) + (1-a)V_H(b, p_2), \\ &= V_H(b, ap_1 + (1-a)p_2), \\ &\leq V(b, ap_1 + (1-a)p_2). \end{aligned} \tag{18}$$

Consequently,  $V(b, ap_1 + (1-a)p_2) = V_H(b, ap_1 + (1-a)p_2)$ ; and hence  $ap_1 + (1-a)p_2 \in \Phi_H^b$ , which proves the convexity of  $\Phi_H^b$ . Since it is always optimal to transmit at rate  $R_2$  if the channel is in a GOOD state (see [17])  $\{1\} \in \Phi_H^b$ , and since the convex subsets of the real line are intervals, there exists  $\rho_N(b) \in (0, 1]$  such that  $\Phi_H^b = [\rho_N(b), 1]$ . Note that  $N$  is the number of thresholds and it is an unknown parameter which depends on the system parameters. Using the same technique we can prove that  $\Phi_{OD}^b$  and  $\Phi_{OT}^b$  are both convex, and hence, there exists  $0 < \rho_{i_1}(b) \leq \rho_{i_2}(b) \leq \rho_{j_1}(b) \leq \rho_{j_2}(b) \leq \rho_N(b) \leq 1$  such that  $\Phi_{OD}^b = [\rho_{i_1}(b), \rho_{i_2}(b)]$  and  $\Phi_{OT}^b = [\rho_{j_1}(b), \rho_{j_2}(b)]$ ; or  $\Phi_{OT}^b = [\rho_{i_1}(b), \rho_{i_2}(b)]$  and  $\Phi_{OD}^b = [\rho_{j_1}(b), \rho_{j_2}(b)]$ . However, since  $V_A(b, ap_1 + (1-a)p_2) \neq aV_A(b, p_1) + (1-a)V_A(b, p_2)$  for  $A \in \{D, L\}$  in general,  $\Phi_D^b$  and  $\Phi_L^b$  are not necessarily convex sets. As a result the remaining segments of the belief set can be partitioned among actions  $D$  and  $L$  in an infinitely many ways. Hence, the number of thresholds,  $N$ , can be infinity. Finding the exact  $N$  and corresponding threshold values is elusive and it is out of the scope of this paper.  $\square$

Although the optimal policy is of threshold-type, as shown in Theorem 1, the subsets of the belief space associated with actions  $D$  and  $L$ , i.e.,  $\Phi_D^b$  and  $\Phi_L^b$ , are not necessarily convex. Each of these sets can be composed of infinitely many intervals; therefore, despite the threshold-type

structure, characterizing the optimal policy may require identifying infinitely many threshold values.

*B. Special Case:  $R_1 = 0$*

In order to further simplify the solution, we assume that it is not possible to transmit any bits when the channel is in a BAD state, i.e.,  $R_1 = 0$  and  $R_2 = R$ . Hence, action  $L$  is no longer available and the action for sensing the channel consists of only  $OD$  which is denoted by  $O$  in the rest of this section.

With this modified model, the expected reward function can be simplified as follows:

$$R(S_t, A_t) = \begin{cases} X_t R, & \text{if } A_t = H \text{ and } B_t \geq 1, \\ (1 - \tau)X_t R, & \text{if } A_t = O \text{ and } B_t \geq 1, \\ 0, & \text{otherwise.} \end{cases} \quad (19)$$

Since at least one energy unit is required for transmission, if the battery state is less than one unit, the reward in (19) becomes zero. If the action of transmitting without sensing is chosen,  $R$  bits are transmitted successfully if the channel is in a GOOD state and 0 bits if the channel is in a BAD state. If the action of sensing is chosen,  $\tau$  fraction of energy unit is spent sensing the channel with the remainder of the energy unit being used for transmission if the channel is sensed to be in a GOOD state. In this case,  $(1 - \tau)R$  bits are transmitted successfully. If the channel is sensed to be in a BAD state, the EH node remains silent in the rest of the time slot. Finally, if the action of deferring the transmission is taken the EH node neither senses the channel nor transmits, so the reward is zero.

Next, we prove that the optimal policy has a threshold-type structure on the belief state with a finite number of thresholds. Note that, in the modified model, the value function is still convex and Lemmas 1, 2 and 3 still hold.

Theorem 2 below states that the optimal solution of the problem defined in (3) is a threshold-type policy with either two or three thresholds on the belief state. The threshold values depend on the state of the battery and system parameters.

**Theorem 2.** *Let  $p \in [0, 1]$  and  $b \geq 0$ . There are thresholds  $0 \leq \rho_1(b) \leq \rho_2(b) \leq \rho_3(b) \leq 1$ , all of which are functions of the battery state  $b$ , such that for  $\tau \leq b < 1$*

$$\pi^*(b, p) = \begin{cases} D, & \text{if } 0 \leq p \leq \rho_1(b) \text{ or } \rho_2(b) \leq p \leq 1, \\ O, & \text{if } \rho_1(b) \leq p \leq \rho_2(b). \end{cases} \quad (20)$$

and for  $b \geq 1$ ,

$$\pi^*(b, p) = \begin{cases} D, & \text{if } 0 \leq p \leq \rho_1(b) \text{ or } \rho_2(b) \leq p \leq \rho_3(b) \\ O, & \text{if } \rho_1(b) \leq p \leq \rho_2(b), \\ H, & \text{if } \rho_3(b) \leq p \leq 1, \end{cases} \quad (21)$$

*Proof.* The proof follows similarly to the proof of Theorem 1. Consider the sets  $\Phi_A^b$  defined in (13) for  $A \in \{D, O, H\}$ . Note that for  $b = 0$ ,  $V(b, p) = V_D(b, p)$ , and hence,  $\Phi_D^0 = [0, 1]$ , and  $\Phi_O^0 = \Phi_H^0 = \emptyset$ . First, consider battery states  $\tau \leq b < 1$ . We prove that for any  $\tau \leq b < 1$ ,  $\Phi_O^b$  is convex, which implies the structure of the optimal policy in (20). Let  $p_1, p_2 \in \Phi_O^b$ , and  $a \in (0, 1)$ . We have

$$V(b, ap_1 + (1 - a)p_2) \leq aV(b, p_1) + (1 - a)V(b, p_2), \quad (22)$$

$$= aV_O(b, p_1) + (1 - a)V_O(b, p_2), \quad (23)$$

$$= V_O(b, ap_1 + (1 - a)p_2), \quad (24)$$

$$\leq V(b, ap_1 + (1 - a)p_2), \quad (25)$$

where (22) follows from Lemma 1; (23) is due to the fact that  $p_1, p_2 \in \Phi_O^b$ ; (24) follows from the linearity of  $V_O$  in  $p$ ; and (25) holds due to the definition of  $V(b, p)$ . Hence,  $V(b, ap_1 + (1 - a)p_2) = V_O(b, ap_1 + (1 - a)p_2)$ , and it follows that  $ap_1 + (1 - a)p_2 \in \Phi_O^b$ , which, in turn, proves the convexity of  $\Phi_O^b$ . Note also that  $p = 0$  and  $p = 1$  both belong to  $\Phi_D^b$  for all  $0 \leq b < 1$ . Hence, for a given battery state  $0 \leq b < 1$ , either  $\Phi_O^b = \emptyset$ , or there exists  $0 < \rho_1(b) \leq \rho_2(b) < 1$  such that  $\Phi_O^b = [\rho_1(b), \rho_2(b)]$ . Consequently, we have  $\Phi_D^b = [0, \rho_1(b)) \cup (\rho_2(b), 1]$ .

Next, consider  $1 \leq b \leq B_{max}$ . We prove that  $\Phi_H^b$  and  $\Phi_O^b$  are both convex, which implies the structure of the optimal policy in (21). Let  $p_1, p_2 \in \Phi_H^b$  and  $a \in (0, 1)$ . Similar to (14)-(17) we can argue

$$\begin{aligned} V(b, ap_1 + (1 - a)p_2) &\leq aV(b, p_1) + (1 - a)V(b, p_2), \\ &= aV_H(b, p_1) + (1 - a)V_H(b, p_2), \\ &= V_H(b, ap_1 + (1 - a)p_2), \\ &\leq V(b, ap_1 + (1 - a)p_2). \end{aligned} \quad (26)$$

Thus,  $V(b, ap_1 + (1 - a)p_2) = V_H(b, ap_1 + (1 - a)p_2)$ ; and hence,  $ap_1 + (1 - a)p_2 \in \Phi_H^b$ , which proves the convexity of  $\Phi_H^b$ . Since it is always optimal to transmit at rate  $R_2$  if the channel is in a GOOD state  $\{1\} \in \Phi_H^b$ , and since the convex subsets of the real line are intervals, there exists  $\rho_3(b) \in (0, 1]$  such that  $\Phi_H^b = [\rho_3(b), 1]$ . Using the same technique we can prove that  $\Phi_O^b$  is convex, and hence, there exists  $0 < \rho_1(b) \leq \rho_2(b) < 1$  such that  $\Phi_O^b = [\rho_1(b), \rho_2(b)]$ . As a result the remaining segments belong to action  $D$ , and we have  $\Phi_D = [0, \rho_1(b)) \cup (\rho_2(b), \rho_3(b))$ .  $\square$

Theorem 2 proves that at any battery state  $b \geq 1$ , at most three threshold values are sufficient to characterize the optimal policy; whereas two thresholds suffice for  $0 \leq b < 1$ . However the optimal policy can even be simpler for some battery states and some instances of the problem as it is possible to have  $\rho_2(b) = \rho_3(b)$ , or even  $\rho_1(b) = \rho_2(b) = \rho_3(b)$ . Since,  $\Phi_D^b$  is not a convex set in general (see Theorem 1), the structure of the optimal policy may result in four different regions even though there are only three possible actions. This may seem counter intuitive for the reader since deferring the transmission should not be advantageous when the belief is relatively high. Nevertheless, in Section V, we demonstrate that in some cases it is indeed optimal to have a three-threshold policy.

## V. NUMERICAL RESULTS

In this section, we use numerical techniques to characterize the optimal policy, and evaluate its performance. We utilize the value iteration algorithm to calculate the optimal value function. We numerically identify the thresholds for the optimal policy for different scenarios. We also evaluate the performance of the optimal policy, and compare it with some alternative policies in terms of throughput.

### A. Evaluating the optimal policy

In the following, we consider the modified system model introduced in Section IV-B in which there is no transmission during BAD channel state, i.e.,  $R_1 = 0$ . We assume that  $B_{max} = 5$ ,  $\tau = 0.2$ ,  $\beta = 0.98$ ,  $\lambda_1 = 0.9$ ,  $\lambda_0 = 0.6$ ,  $R = 3$  and  $q = 0.1$ . The optimal policy is evaluated using the value iteration algorithm. In Fig. 2, each state  $(b, p)$  is illustrated with a different color corresponding to the optimal policy at that state. In Fig. 2, the areas highlighted with blue color correspond to those states at which deferring the transmission is optimal, green areas correspond to the states at which sensing the channel is optimal, and finally yellow areas correspond to the states for which transmitting at high rate is optimal. As seen in Fig. 2, any of the three

policies (one, two, or three threshold policies) may be optimal depending on the battery state. For example, when the battery state is  $b = 2$ , one-threshold policy is optimal. The transmitter defers transmission up to a belief state of  $p = 0.8$ , and starts transmitting without sensing beyond this value. For no value of the belief state it opts for sensing the channel. On the other hand, when the battery state is 3.8, two-threshold policy is optimal, and when the battery state is 2.8, three-threshold policy is optimal. Considering the low probability of energy arrivals ( $q = 0.1$ ) and the relative high cost of sensing ( $\tau = 0.2$ ), the transmitter senses the channel even when its battery state is below the transmission threshold, i.e.,  $b < 1$ .

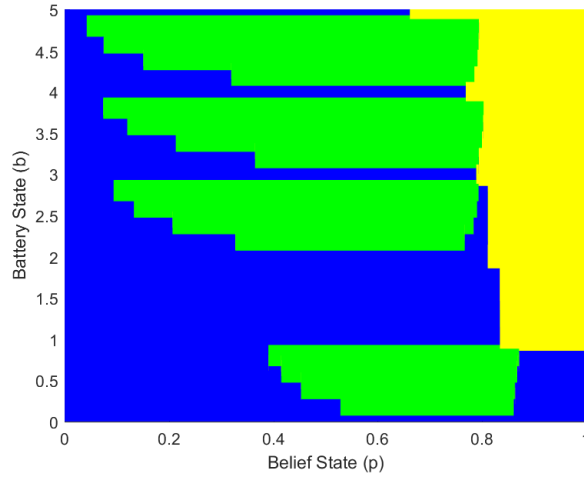


Fig. 2: Optimal thresholds for taking the actions D (blue), O (green), H (yellow) for  $B_{max} = 5$ ,  $\tau = 0.2$ ,  $\beta = 0.98$ ,  $\lambda_1 = 0.9$ ,  $\lambda_0 = 0.6$ ,  $R = 3$  and  $q = 0.1$ .

Another interesting result is the periodicity in the behavior of the optimal policy with respect to the battery state observed in Fig. 2. This is particularly observed when action  $D$  is taken when the battery state takes integer values, which is then followed by action  $O$  for increasing beliefs when the battery state is more than 2. The value function corresponding to the parameters used to obtain Fig. 2 is depicted in Fig. 3. Note the staircase behavior of the value function. There is a jump in the value function at the integer values of the battery state, while it approximately remains the same when the battery state is confined between two consecutive integer numbers. Hence, when the battery state of the transmitter is an integer, any action other than deferring will, with high probability, transition into a state with a relatively lower value. Thus, the transmitter chooses action  $D$  unless its belief is relatively high. However, when the battery state is between



two consecutive integer numbers, it is safe to sense the channel since in the worst case the channel is in a BAD state and the transmitter loses only  $\tau < 1$  units, but it makes a transition into a state which approximately has the same value. Thus at those values of the battery, the transmitter chooses to sense the channel for moderate belief states.

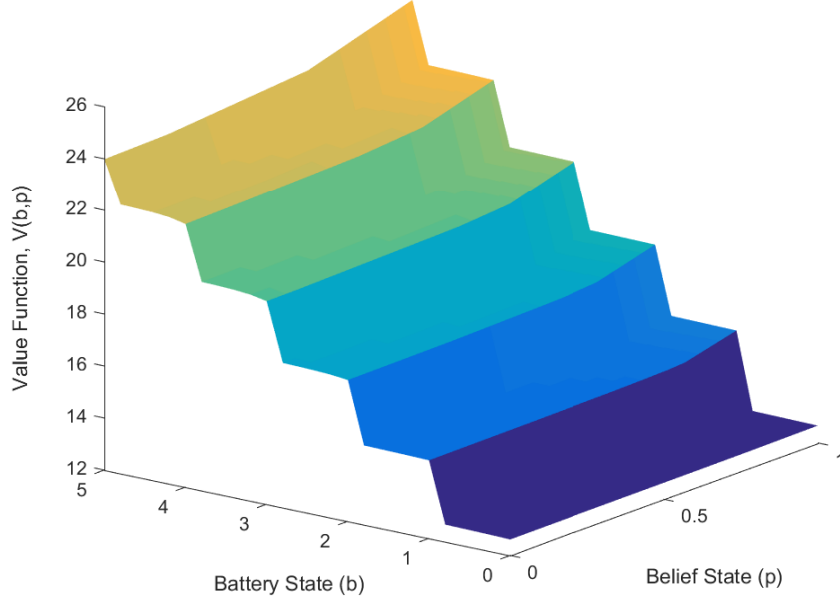


Fig. 3: Value function associated with  $B_{max} = 5$ ,  $\tau = 0.2$ ,  $\beta = 0.98$ ,  $\lambda_1 = 0.9$ ,  $\lambda_0 = 0.6$ ,  $R = 3$  and  $q = 0.1$ .

To investigate the effect of the EH rate,  $q$ , on the optimal transmission policy, we consider the system parameters  $B_{max} = 5$ ,  $\tau = 0.1$ ,  $\beta = 0.9$ ,  $\lambda_1 = 0.8$ ,  $\lambda_0 = 0.4$ , and  $R = 3$ . We illustrate the optimal transmission policy for  $q = 0.8$  and  $q = 0.2$  in Fig. 4a and Fig. 4b, respectively. It can be observed by comparing those two figures that the yellow regions are much larger and blue areas are much more limited in Fig. 4a. This is because when the energy arrivals are more frequent, the EH node tends to consume its energy more generously. We also observe that the EH node always defers its transmission for  $b < 1$  when energy is limited (in Fig. 4b), whereas it may opt for sensing the channel when energy is more abundant.

Next, we investigate the effect of the sensing cost,  $\tau$ , on the optimal policy. To illustrate this effect, we choose the system parameters as  $B_{max} = 5$ ,  $\beta = 0.9$ ,  $\lambda_1 = 0.8$ ,  $\lambda_0 = 0.4$ ,  $R = 3$  and  $q = 0.8$ . The regions for optimal actions are shown in Fig. 5a and Fig. 5b for sensing cost

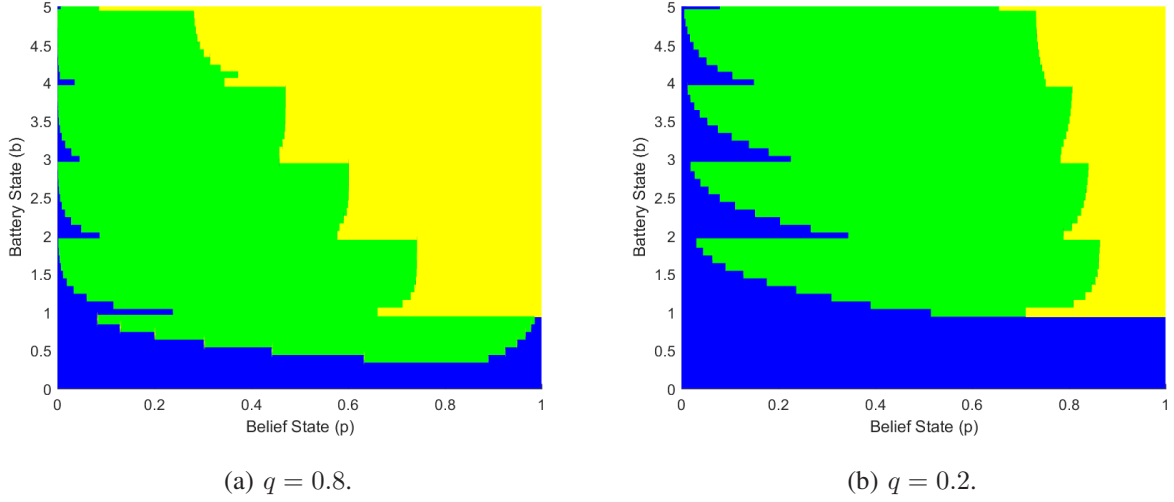


Fig. 4: Optimal thresholds for taking the actions D (blue), O (green), H (yellow) for  $B_{max} = 5$ ,  $\tau = 0.1$ ,  $\beta = 0.9$ ,  $\lambda_1 = 0.8$ ,  $\lambda_0 = 0.4$ , and  $R = 3$ .

values  $\tau = 0.2$  and  $\tau = 1/3$ , respectively. By comparing Fig. 5a and Fig. 5b, it is evident that a higher cost of sensing results in a less incentive for sensing the channel. We observe in Fig. 5b that the green areas have shrunk as compared to Fig. 5a, i.e., the transmitter is more likely to take a risk and transmit without sensing, or defer its transmission, when sensing consumes a significant portion of the available energy.

### B. Throughput performance

In this section, we compare the performance of the optimal policy with three alternative policies, i.e., a greedy policy, a single-threshold policy and an opportunistic policy. In the greedy policy, the EH node transmits whenever it has energy in its battery. In the single-threshold policy, there are only two actions: defer (D) or transmit (H). The belief of the transmitter on the current channel state depends only on the ACK/NACK feedback from the receiver, and channel sensing is not exploited at all. We optimize the threshold corresponding to each battery state for the single-threshold policy using the value iteration algorithm. Meanwhile, the opportunistic policy senses the channel at the beginning of every time slot, and transmits  $(1 - \tau)R$  bits if the channel is in a GOOD state, and it defers otherwise. By choosing the parameters  $B_{max} = 5$ ,  $\beta = 0.999$ ,  $\lambda_1 = 0.8$ ,  $\lambda_0 = 0.2$ ,  $R = 2$ , the throughput achieved by these four policies are plotted in Fig.

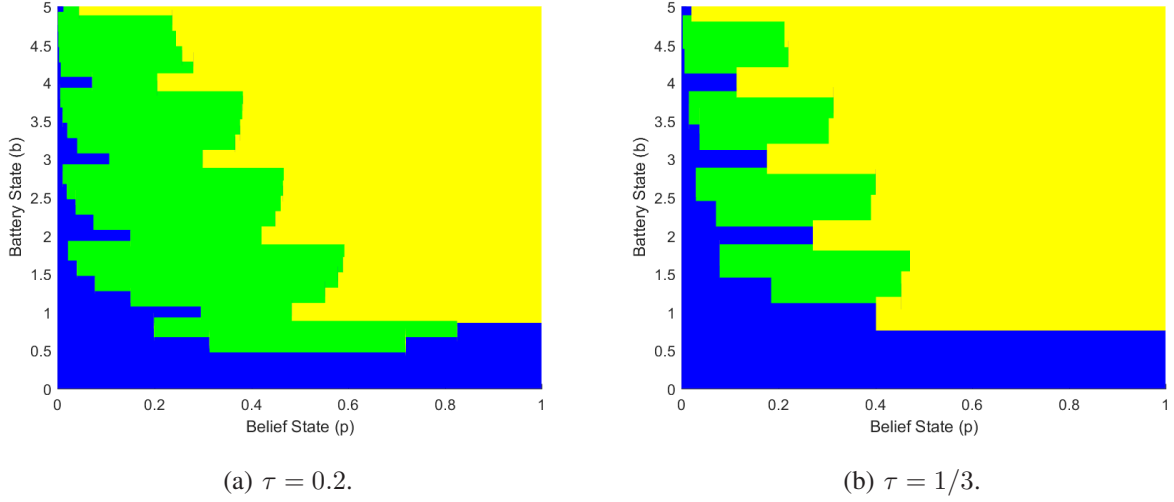


Fig. 5: Optimal thresholds for taking the actions D (blue), O (green), H (yellow) for  $B_{max} = 5$ ,  $\beta = 0.9$ ,  $\lambda_1 = 0.8$ ,  $\lambda_0 = 0.4$ ,  $R = 3$  and  $q = 0.8$ .

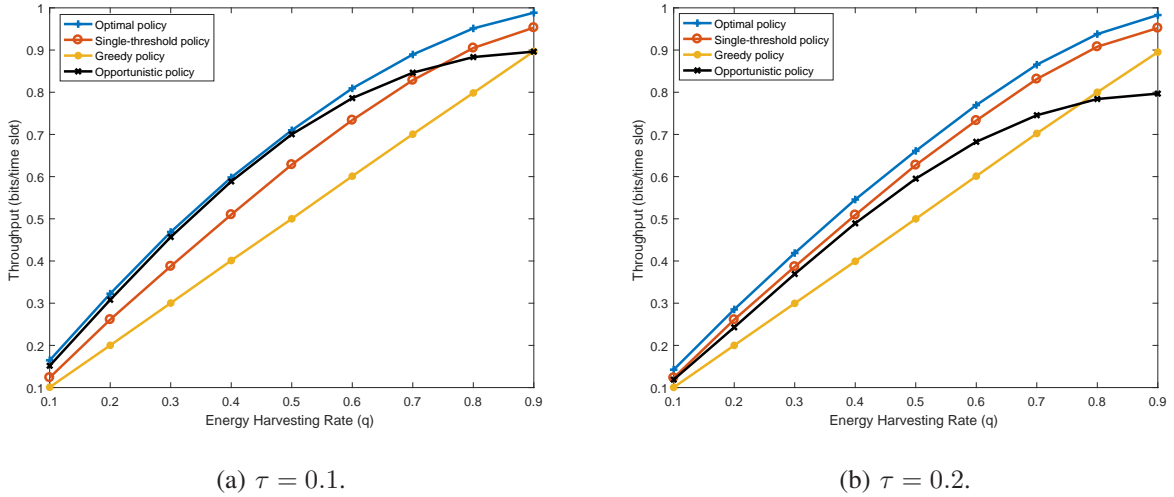


Fig. 6: Throughputs by the optimal, greedy, single-threshold and opportunistic policies as a function of the EH rate,  $q$ .

6 with respect to the EH rate  $q$ . Fig. 6a and 6b correspond to the sensing costs of  $\tau = 0.1$  and  $\tau = 0.2$ , respectively.

As expected, we observe that the greedy policy performs much worse than the optimal policy as it does not exploit the transmitter's knowledge about the state of the channel. We can see that, by simply exploiting the ACK/NACK feedback from the receiver in order to defer the

transmission, the single-threshold policy achieves a higher throughput than the greedy policy at all values of the EH rate. Note that single-threshold and greedy policies do not have the sensing capability, and accordingly, the sensing cost,  $\tau$ , has no effect on their performance. However,  $\tau$  affects the optimal and opportunistic policies which have sensing capabilities. In particular,  $\tau$  affects the opportunistic policy drastically, since this policy senses the channel at the beginning of each time slot. When the sensing cost is relatively low, it can be seen from Fig. 6a that opportunistic policy achieves a near optimal throughput except when the EH rate,  $q$ , is high. For high values of  $q$ , the EH transmitter suffers less from energy deprivations and instead of sensing at each time slot, using the whole time slot for transmission becomes more beneficial. Hence, we observe that always sensing the channel is not a good idea when the energy arrival rate  $q$  is high as seen in Fig. 6a and 6b. When the sensing cost  $\tau$  is relatively high, it can be seen from Fig. 6b that opportunistic policy performs worse than the single-threshold policy for all values of  $q$ , and even worse than the greedy policy for high values of  $q$ . On the other hand, the optimal policy by intelligently utilizing the sensing capability yields a superior performance for all the parameter values.

### C. Optimal policy evaluation with two different transmission rates

When the transmitter has the ability to transmit at two different rates, we proved that the optimal policy is a threshold-type policy. However, due to non-convexity of sets  $\Phi_D^b$  and  $\Phi_L^b$  it is not possible to characterize the optimal policy as we have done for a transmitter with a single rate in (21) and (20). Instead, we numerically evaluate the optimal policy as follows.

Let  $B_{max} = 5$ ,  $\tau = 1/28$ ,  $\beta = 0.7$ ,  $\lambda_1 = 0.98$ ,  $\lambda_0 = 0.81$ ,  $R_1 = 2.91$ ,  $R_2 = 3$  and  $q = 0.1$ . Note that these parameters are chosen in a way to show the non-convexity of the sets  $\Phi_D^b$  and  $\Phi_L^b$  and they may not exist in a practical scenario. The optimal policy, obtained through the value iteration algorithm, is represented in Fig. 7. In the figure, the areas highlighted with blue color correspond to those states at which deferring ( $D$ ) the transmission is optimal, red color correspond to those states at which transmitting at the low rate ( $L$ ) is optimal, green color correspond to those states in which sensing and deferring is optimal ( $OD$ ), black color correspond to the states at which sensing and transmitting opportunistically ( $OT$ ) is optimal, and yellow color correspond to the states for which transmitting without sensing ( $H$ ) is optimal.

As expected the optimal policy is again a battery-dependent threshold-type policy with respect to the belief state. The sets  $\Phi_D^b$  and  $\Phi_L^b$  (blue and red areas, respectively) are not convex. In

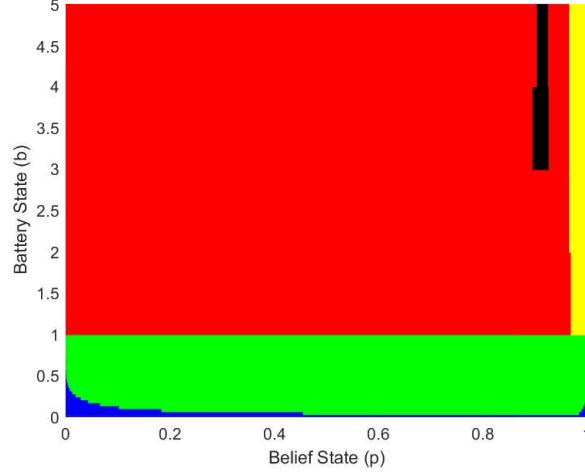


Fig. 7: Optimal thresholds for taking the actions D (blue), L (red), OD (green), OT (black), H (yellow) for  $B_{max} = 5$ ,  $\tau = 1/28$ ,  $\beta = 0.7$ ,  $\lambda_1 = 0.98$ ,  $\lambda_0 = 0.81$ ,  $R_1 = 2.91$ ,  $R_2 = 3$  and  $q = 0.1$ .

theory, an optimal policy may have infinite threshold values if the sets  $\Phi_D^b$  and  $\Phi_L^b$  are intertwined into infinitely many alternating intervals. We observe in Fig. 7 that, for the parameters considered here, this is not the case and the optimal policy consists of at most three-threshold policies.

## VI. CONCLUSIONS AND FUTURE WORK

In this work, we considered an EH transmitter equipped with a battery, operating over a time-varying finite-capacity wireless channel with memory, modeled as a two-state Gilbert-Elliot channel. The transmitter receives ACK/NACK feedback after each transmission, which can be used to track the channel state. Additionally, the transmitter has the capability to sense the channel, which allows the transmitter to obtain the current channel state at a certain energy and time cost. Therefore, at the beginning of each time slot, the transmitter has the following possible actions to maximize the total expected discounted number of bits transmitted over an infinite time horizon: *i*) deferring the transmission to save its energy for future use, *ii*) transmitting at a low rate of  $R_1$  bits with guaranteed successful delivery of the message, *iii*) transmitting at a high rate of  $R_2$  bits, and *iv*) sensing the channel to reveal the channel state by consuming a portion of its energy and transmission time, and then deciding either to defer or to transmit at a suitable rate based on the revealed channel state. We formulated the problem as a POMDP, which is then converted into a MDP with continuous state space by introducing a belief parameter for

the channel state. We have shown that the optimal transmission policy has a threshold structure with respect to the belief state, where the optimal threshold values depend on the battery state.

We then considered the simplified problem by assuming that it is not possible to transmit any information when the channel is in a BAD state, for which we were able to prove that the optimal policy has at most three threshold values. We calculated the optimal threshold values numerically using the value iteration algorithm. We compared the throughput achieved by the optimal policy to those achieved by a greedy policy and a single-threshold policy, both of which do not exploit the channel sensing capability, as well as an opportunistic policy, which senses the channel at every time slot. We have shown through simulations that the intelligent channel sensing capability improves the performance significantly, thanks to the increased adaptability to the channel conditions. As a future work, we will consider the effect of imperfect channel sensing.

## APPENDIX A

### PROOF OF LEMMA 1

*Proof.* Define  $V(b, p, n)$  as the optimal value function for the finite-horizon problem spanning only  $n$  time slots. We will first prove the convexity of  $V(b, p, n)$  in  $p$  by induction. Optimal value function can be written as follows,

$$V(b, p, n) = \max \{V_D(b, p, n), V_L(b, p, n), V_{OD}(b, p, n), V_{OT}(b, p, n), V_H(b, p, n)\}, \quad (27)$$

where

$$V_D(b, p, n) = \beta [qV(\min(b+1, B_{max}), J(p), n-1) + (1-q)V(b, J(p), n-1)], \quad (28)$$

$$V_L(b, p, n) = R_1 + \beta [qV(b, J(p), n-1) + (1-q)V(b-1, J(p), n-1)], \quad (29)$$

$$\begin{aligned} V_{OD}(b, p) = & p[(1-\tau)R_2 + \beta(qV(b, \lambda_1) + (1-q)V(b-1, \lambda_1))] \\ & + (1-p)\beta[qV(\min\{b-\tau+1, B_{max}\}, \lambda_0) + (1-q)V(b-\tau, \lambda_0)], \end{aligned} \quad (30)$$

$$\begin{aligned} V_{OT}(b, p) = & p[(1-\tau)R_2 + \beta(qV(b, \lambda_1) + (1-q)V(b-1, \lambda_1))] \\ & + (1-p)[(1-\tau)R_1 + \beta(qV(b, \lambda_0) + (1-q)V(b-1, \lambda_0))] \quad b \geq 1, \end{aligned} \quad (31)$$

$$\begin{aligned} V_{OD}(b, p, n) = & V_{OT}(b, p, n) = \beta [qpV(b-\tau+1, \lambda_1, n-1) \\ & + q(1-p)V(b-\tau+1, \lambda_0, n-1) + (1-q)pV(b-\tau, \lambda_1, n-1) \\ & + (1-q)(1-p)V(b-\tau, \lambda_0, n-1)], \quad \text{for } \tau \leq b < 1, \end{aligned} \quad (32)$$

$$V_H(b, p, n) = p[R + \beta(qV(b, \lambda_1, n-1) + (1-q)V(b-1, \lambda_1, n-1))] \\ + (1-p)\beta[V(b, \lambda_0, n-1) + (1-q)V(b-1, \lambda_0, n-1)], \quad b \geq 1. \quad (33)$$

Note that when  $b < 1$ , we have  $V(b, p, 1) = 0$ , and when  $b \geq 1$  we have  $V(b, p, 1) = \max\{R_1, pR_2, (1-\tau)pR_2, (1-\tau)[pR_2 + (1-p)R_1]\}$  which is a maximum of four convex functions. We see that  $V(b, p, 1)$  is a convex function of  $p$ .

Now, let us assume that  $V(b, p, n-1)$  is convex in  $p$  for any  $b \geq 0$ , then for  $a \in [0, 1]$  we can investigate the convexity of the value function for each action separately as follows.

For deferring the transmission, i.e.,  $A = D$ , we can write:

$$V_D(b, ap_1 + (1-a)p_2, n) = \beta[qV(\min(b+1, B_{max}), J(ap_1 + (1-a)p_2), n-1) \\ + (1-q)V(b, J(ap_1 + (1-a)p_2), n-1)] \\ = \beta[qV(\min(b+1, B_{max}), aJ(p_1) + (1-a)J(p_2), n-1) \\ + (1-q)V(b, aJ(p_1) + (1-a)J(p_2), n-1)] \\ \leq a\beta[qV(\min(b+1, B_{max}), J(p_1), n-1) \\ + (1-q)V(b, J(p_1), n-1)] \\ + (1-a)\beta[qV(\min(b+1, B_{max}), J(p_2), n-1) \\ + (1-q)V(b, J(p_2), n-1)] \\ = aV_D(b, p_1, n) + (1-a)V_D(b, p_2, n) \\ \leq aV(b, p_1, n) + (1-a)V(b, p_2, n) \quad (34)$$

Hence,  $V_D(b, p, n)$  is convex in  $p$ . Similarly let us consider the action  $L$ .

$$V_L(b, ap_1 + (1-a)p_2, n) = R_1 + \beta[qV(b, J(ap_1 + (1-a)p_2), n-1) \\ + (1-q)V(b-1, J(ap_1 + (1-a)p_2), n-1)] \\ = R_1 + \beta[qV(b, aJ(p_1) + (1-a)J(p_2), n-1) \\ + (1-q)V(b-1, aJ(p_1) + (1-a)J(p_2), n-1)] \\ \leq aR_1 + a\beta[qV(b, J(p_1), n-1) \\ + (1-q)V(b-1, J(p_1), n-1)]$$

$$\begin{aligned}
& + (1 - a)R_1 + (1 - a)\beta [ qV(b, J(p_2), n - 1) \\
& + (1 - q)V(b - 1, J(p_2), n - 1) ] \\
& = aV_D(b, p_1, n) + (1 - a)V_D(b, p_2, n) \\
& \leq aV(b, p_1, n) + (1 - a)V(b, p_2, n)
\end{aligned} \tag{35}$$

Thus,  $V_L(b, p, n)$  is also convex in  $p$ . Note that  $V_{OD}(b, p, n)$ ,  $V_{OT}(b, p, n)$ , and  $V_H(b, p, n)$  are linear functions of  $p$ , thus they are also convex in  $p$ .

Since the value function  $V(b, p, n)$  is the maximum of five (or, in some cases two) convex functions when  $b \geq 1$  ( $\tau \leq b < 1$ ), it is also convex.

By induction we can claim the convexity of  $V(b, p, n)$  for all  $n$ . Since  $V(b, p, n) \rightarrow V(b, p)$  as  $n \rightarrow \infty$ ,  $V(b, p)$  is also convex.  $\square$

## APPENDIX B

### PROOF OF LEMMA 2

*Proof.* We will again use induction to prove the claim for  $V(b, p, n)$  defined as in Appendix A as the optimal value function when the decision horizon spans  $n$  stages. We have  $V(b, p, 1) = 0$  if  $b < 1$  and we have  $V(b, p, 1) = \max \{R_1, pR_2, (1 - \tau)pR_2, (1 - \tau)[pR_2 + (1 - p)R_1]\}$  if  $b \geq 1$ . Hence,  $V(b, p, 1)$  is trivially non-decreasing in  $b$ . Suppose that  $V(b, p, n - 1)$  is non-decreasing in  $b$ . Each of the value functions given in (28), (29), (30), (31), (32) and (33) is the summation of positive weighted non-decreasing functions. Therefore, they are all non-decreasing in  $b$ . Since the optimal value function is the maximum of these non-decreasing functions, it is also non-decreasing in  $b$  for any  $n$ . Similarly to Appendix A, by letting  $n \rightarrow \infty$ , we conclude that  $V(b, p)$  is non-decreasing in  $b$ .  $\square$

## APPENDIX C

### PROOF OF LEMMA 3

*Proof.* We employ induction on  $V(b, p, n)$  once again. For  $n = 1$ ,  $V(b, p, 1)$  is 0 if  $b < 1$ , and  $\max \{R_1, pR_2, (1 - \tau)pR_2, (1 - \tau)[pR_2 + (1 - p)R_1]\}$  if  $b \geq 1$ . Therefore,  $V(b, p, 1)$  is non-decreasing in  $p$  for any given  $b$ .

Assume that  $V(b, p, n - 1)$  is non-decreasing in  $p$ . Since  $J(p)$  is non-decreasing, it is easy to see that  $V_D(b, p, n)$  and  $V_L(b, p, n)$  respectively in (28) and (29) are also non-decreasing.



Since  $V_A(b, p, n)$ s for  $A \in \{OD, OT, H\}$  are linear in  $p$ , we have  $V_A(b, ap_1 + (1-a)p_0, n) = aV_A(b, p_1, n) + (1-a)V_A(b, p_0, n)$ . Using this result, we have

$$V_A(b, p_1, n) - V_A(b, p_0, n) = V_A(b, p_1 - p_0 + p_0, n) - V_A(b, p_0, n) \quad (36a)$$

$$= V_A(b, p_1 - p_0, n) \geq 0 \quad A \in \{OD, OT, H\} \quad (36b)$$

Note that (36b) follows from the fact that  $V_A(b, p_1 - p_0 + p_0, n) = V_A(b, p_1 - p_0, n) + V_A(b, p_0, n)$ . Since the value function,  $V(b, p, n)$ , is the maximum of non-decreasing functions, it is also non-decreasing. Hence, by letting  $n \rightarrow \infty$ , we prove that  $V(b, p)$  is non-decreasing in  $p$ .  $\square$

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